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# PION VELOCITY NEAR THE CHIRAL PHASE TRANSITION \*

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We study the pion velocity near the critical temperature  $T_c$  of chiral symmetry restoration in QCD. Using the hidden local symmetry (HLS) model as the effective field theory, where the chiral symmetry restoration is realized as the vector manifestation (VM), we show that the pion velocity for  $T \rightarrow T_c$  receives neither quantum nor (thermal) hadronic corrections at the critical temperature even when we start from the bare theory with Lorentz symmetry breaking. We show that this is related to a new fixed point structure originated in the VM. Further we match at a matching scale the axial-vector current correlator in the HLS with the one in the operator product expansion for QCD, and present the matching condition to determine the bare pion velocity. We find that the pion velocity is close to the speed of light,  $v_\pi(T) = 0.83 - 0.99$ .

## 1. Introduction

Chiral symmetry in QCD is expected to be restored under some extreme conditions such as large number of flavor  $N_f$  and high temperature and/or density. In hadronic sector, the chiral symmetry restoration is described by various effective field theories (EFTs) based on the chiral symmetry <sup>3</sup>.

By using the hidden local symmetry (HLS) model <sup>4</sup> as an EFT and performing the Wilsonian matching which is one of the methods that determine the bare theory from the underlying QCD <sup>5</sup>, the vector manifestation (VM) in hot or dense matter was formulated in Refs. 6, 7. In the VM, the massless vector meson becomes the chiral partner of pion at the critical point <sup>8 a</sup>. There, the *intrinsic temperature or density dependences of the parameters* of the HLS Lagrangian, which are obtained by integrating out the high

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<sup>a</sup>As studied in Ref. 15 in detail, the VM is defined only as a limit with bare parameters approaching the VM fixed point from the broken phase.

energy modes (i.e., the quarks and gluons above the matching scale) in hot and/or dense matter, play the essential roles to realize the chiral symmetry restoration consistently. That the vector meson mass vanishes at the critical temperature/density supports the in-medium scaling of the vector meson proposed by Brown and Rho, Brown-Rho scaling<sup>9</sup>, and has qualitatively important influences on the properties of hadrons in medium.

In the analysis done in Ref. 10, it was shown that the effect of Lorentz symmetry breaking to the bare parameters caused by the intrinsic temperature dependence through the Wilsonian matching are small<sup>10,11</sup>. Starting from the bare Lagrangian with Lorentz invariance, it was presented that the pion velocity approaches the speed of light at the critical temperature<sup>10</sup>, although in low temperature region ( $T \ll T_c$ ) the pion velocity deviates from the speed of light due to hadronic corrections<sup>11</sup>.

However there do exist the Lorentz non-invariant effects in bare EFT anyway due to the intrinsic temperature and/or density effects. Further the Lorentz non-invariance might be enhanced through the renormalization group equations (RGEs), even if effects of Lorentz symmetry breaking at the bare level are small. Thus it is important to investigate how Lorentz non-invariance at bare level influences physical quantities.

In this talk, we pick up the pion velocity at the critical temperature and study the quantum and hadronic thermal effects based on the VM. The pion velocity is one of the important quantities since it controls the pion propagation in medium through a dispersion relation. We show the non-renormalization property on the pion velocity  $v_\pi$ , which is protected by the VM, and estimate the value of  $v_\pi$  near the critical temperature.

## 2. Model Based on the Hidden Local Symmetry

In this section, we show the HLS Lagrangian at leading order including the effects of Lorentz non-invariance.

The HLS model is based on the  $G_{\text{global}} \times H_{\text{local}}$  symmetry, where  $G = SU(N_f)_L \times SU(N_f)_R$  is the chiral symmetry and  $H = SU(N_f)_V$  is the HLS. The basic quantities are the HLS gauge boson  $V_\mu$  and two matrix valued variables  $\xi_L(x)$  and  $\xi_R(x)$  which transform as  $\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g_{L,R}^\dagger$ , where  $h(x) \in H_{\text{local}}$  and  $g_{L,R} \in [SU(N_f)_{L,R}]_{\text{global}}$ . These variables are parameterized as<sup>b</sup>  $\xi_{L,R}(x) = e^{i\sigma(x)/F_\sigma^t} e^{\mp i\pi(x)/F_\pi^t}$ , where

<sup>b</sup>The wave function renormalization constant of the pion field is given by the temporal component of the pion decay constant<sup>1,12</sup>. Thus we normalize  $\pi$  and  $\sigma$  by  $F_\pi^t$  and  $F_\sigma^t$

$\pi = \pi^a T_a$  denotes the pseudoscalar Nambu-Goldstone bosons associated with the spontaneous symmetry breaking of  $G_{\text{global}}$  chiral symmetry, and  $\sigma = \sigma^a T_a$  denotes the Nambu-Goldstone bosons associated with the spontaneous breaking of  $H_{\text{local}}$ . This  $\sigma$  is absorbed into the HLS gauge boson through the Higgs mechanism, and then the vector meson acquires its mass.  $F_\pi^t$  and  $F_\sigma^t$  denote the temporal components of the decay constant of  $\pi$  and  $\sigma$ , respectively. The covariant derivative of  $\xi_L$  is given by

$$D_\mu \xi_L = \partial_\mu \xi_L - iV_\mu \xi_L + i\xi_L \mathcal{L}_\mu, \quad (1)$$

and the covariant derivative of  $\xi_R$  is obtained by the replacement of  $\mathcal{L}_\mu$  with  $\mathcal{R}_\mu$  in the above where  $V_\mu$  is the gauge field of  $H_{\text{local}}$ , and  $\mathcal{L}_\mu$  and  $\mathcal{R}_\mu$  are the external gauge fields introduced by gauging  $G_{\text{global}}$  symmetry. In terms of  $\mathcal{L}_\mu$  and  $\mathcal{R}_\mu$ , we define the external axial-vector and vector fields as  $\mathcal{A}_\mu = (\mathcal{R}_\mu - \mathcal{L}_\mu)/2$  and  $\mathcal{V}_\mu = (\mathcal{R}_\mu + \mathcal{L}_\mu)/2$ .

In the HLS model it is possible to perform the derivative expansion systematically<sup>13,14,15</sup>. In the chiral perturbation theory (ChPT) with HLS, the vector meson mass is to be considered as small compared with the chiral symmetry breaking scale  $\Lambda_\chi$ , by assigning  $\mathcal{O}(p)$  to the HLS gauge coupling,  $g \sim \mathcal{O}(p)$ <sup>13,14</sup>. (For details of the ChPT with HLS, see Ref. 15.) The leading order Lagrangian with Lorentz non-invariance can be written as<sup>7</sup>

$$\begin{aligned} \mathcal{L} = & \left[ (F_\pi^t)^2 u_\mu u_\nu + F_\pi^t F_\pi^s (g_{\mu\nu} - u_\mu u_\nu) \right] \text{tr} [\hat{\alpha}_\perp^\mu \hat{\alpha}_\perp^\nu] \\ & + \left[ (F_\sigma^t)^2 u_\mu u_\nu + F_\sigma^t F_\sigma^s (g_{\mu\nu} - u_\mu u_\nu) \right] \text{tr} [\hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu] \\ & + \left[ -\frac{1}{g_L^2} u_\mu u_\alpha g_{\nu\beta} - \frac{1}{2g_T^2} (g_{\mu\alpha} g_{\nu\beta} - 2u_\mu u_\alpha g_{\nu\beta}) \right] \text{tr} [V^{\mu\nu} V^{\alpha\beta}], \quad (2) \end{aligned}$$

where

$$\hat{\alpha}_{\perp,\parallel}^\mu = \frac{1}{2i} [D^\mu \xi_R \cdot \xi_R^\dagger \mp D^\mu \xi_L \cdot \xi_L^\dagger]. \quad (3)$$

Here  $F_\pi^s$  denote the spatial pion decay constant and similarly  $F_\sigma^s$  for the  $\sigma$ . The rest frame of the medium is specified by  $u^\mu = (1, \vec{0})$  and  $V_{\mu\nu}$  is the field strength of  $V_\mu$ .  $g_L$  and  $g_T$  correspond in medium to the HLS gauge coupling  $g$ . The parametric  $\pi$  and  $\sigma$  velocities are defined by<sup>16</sup>

$$V_\pi^2 = F_\pi^s / F_\pi^t, \quad V_\sigma^2 = F_\sigma^s / F_\sigma^t. \quad (4)$$

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respectively.

### 3. Vector Manifestation Conditions

In this section, we start from the HLS Lagrangian with Lorentz non-invariance, and requiring that the axial-vector current correlator be equal to the vector current correlator at the critical point, we present the conditions satisfied at the critical point.

Concept of the matching in the Wilsonian sense is based on the following assumptions: The bare Lagrangian of the effective field theory (EFT)  $\mathcal{L}_{\text{bare}}$  is defined at a suitable matching scale  $\Lambda$ . Generating functional derived from  $\mathcal{L}_{\text{bare}}$  leads to the same Green's function as that derived from the generating functional of QCD at  $\Lambda$ . In other words, the bare parameters are obtained after integrating out the high energy modes, i.e., the quarks and gluons above  $\Lambda$ . When we integrate out the high energy modes in hot matter, the bare parameters have a certain temperature dependence, *intrinsic temperature dependence*, converted from QCD to the EFT. The intrinsic temperature dependence is nothing but the signature that hadrons have an internal structure constructed from quarks and gluons. In the following, we describe the chiral symmetry restoration based on the point of view that *the bare HLS theory is defined from the underlying QCD*. We note that the Lorentz non-invariance appears in bare HLS theory as a result of including the intrinsic temperature dependences. Once the temperature dependence of the bare parameters is determined through the matching with QCD mentioned above, from the RGEs the parameters appearing in the hadronic corrections pick up the intrinsic thermal effects.

Now we consider the matching near the critical temperature. The axial-vector and vector current correlators at bare level are constructed in terms of bare parameters and are expanded in terms of the longitudinal and transverse projection operators  $P_{L,T}^{\mu\nu}$ :  $G_{A,V}^{\mu\nu} = P_L^{\mu\nu} G_{A,V}^L + P_T^{\mu\nu} G_{A,V}^T$ . At the chiral phase transition point, the axial-vector and vector current correlators must agree with each other, i.e., chiral symmetry restoration is characterized by the following conditions:  $G_{A(\text{HLS})}^L - G_{V(\text{HLS})}^L \rightarrow 0$  and  $G_{A(\text{HLS})}^T - G_{V(\text{HLS})}^T \rightarrow 0$  for  $T \rightarrow T_c$ . In Ref. 7, it was shown that they are satisfied for any values of  $p_0$  and  $\bar{p}$  around the matching scale only if the following conditions are met:  $(g_{L,\text{bare}}, g_{T,\text{bare}}, a_{\text{bare}}^t, a_{\text{bare}}^s) \rightarrow (0, 0, 1, 1)$  for  $T \rightarrow T_c$ . This implies that at bare level the longitudinal mode of the vector meson becomes the real NG boson and couples to the vector current correlator, while the transverse mode decouples.

In Ref. 1, we have shown that  $(g_L, a^t, a^s) = (0, 1, 1)$  is a fixed point of the RGEs and satisfied in any energy scale. Thus the VM condition is given

by

$$(g_L, a^t, a^s) \rightarrow (0, 1, 1) \quad \text{for } T \rightarrow T_c. \quad (5)$$

The vector meson mass is never generated at the critical temperature since the quantum correction to  $M_\rho^2$  is proportional to  $g_L^2$ . Because of  $g_L \rightarrow 0$ , the transverse vector meson at the critical point, in any energy scale, decouples from the vector current correlator. The VM condition for  $a^t$  and  $a^s$  leads to the equality between the  $\pi$  and  $\sigma$  (i.e., longitudinal vector meson) velocities:

$$(V_\pi/V_\sigma)^4 = (F_\pi^s F_\sigma^t / F_\sigma^s F_\pi^t)^2 = a^t/a^s \xrightarrow{T \rightarrow T_c} 1. \quad (6)$$

This is easily understood from a point of view of the VM since the longitudinal vector meson becomes the chiral partner of pion. We note that this condition  $V_\sigma = V_\pi$  holds independently of the value of the bare pion velocity which is to be determined through the Wilsonian matching.

#### 4. Non-renormalization Property on the Pion Velocity

As we have seen in the previous section, the dropping mass of vector meson can be realized as the VM which is formulated by using the HLS theory. Then what is the prediction of the VM? Recently it was proven that the non-renormalization property on the pion velocity which is protected by the VM<sup>1</sup>. In the following, we show that this can be understood based on the idea of chiral partners.

Before going to the critical temperature  $T_c$ , let us consider the situation away from  $T_c$ . Starting from the bare pion velocity  $V_{\pi,\text{bare}}^2 = F_{\pi,\text{bare}}^s / F_{\pi,\text{bare}}^t$  and including quantum and hadronic corrections into the parameters through the diagrams shown in Fig. 1. Away from  $T_c$ , there exists the

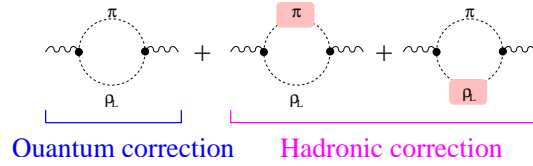


Figure 1. Diagrams for contribution to the pion velocity. Center (right) diagram denotes the interaction between the pion (vector meson) and heat bath.

hadronic thermal correction to the pion velocity<sup>11</sup>:

$$v_\pi^2(T) \simeq V_\pi^2 - N_f \frac{2\pi^2}{15} \frac{T^4}{(F_\pi^t)^2 M_\rho^2} \quad \text{for } T < T_c, \quad (7)$$

where the contribution of the massive  $\sigma$  (i.e., the longitudinal mode of massive vector meson) is suppressed owing to the Boltzmann factor  $\exp[-M_\rho/T]$ , and then only the pion loop contributes to the pion velocity.

On the other hand, when we approach the critical temperature, the vector meson mass goes to zero due to the VM. Thus  $\exp[-M_\rho/T]$  is no longer the suppression factor. As a result, the hadronic correction in the pion velocity is absent due to the exact cancellation between the contribution of pion and that of its chiral partner  $\sigma$ . Similarly the quantum correction generated from the pion loop is exactly cancelled by that from the  $\sigma$  loop. Accordingly we conclude

$$v_\pi(T) = V_{\pi,\text{bare}}(T) \quad \text{for } T \rightarrow T_c, \quad (8)$$

i.e., *the pion velocity in the limit  $T \rightarrow T_c$  receives neither hadronic nor quantum corrections due to the protection by the VM.* This implies that  $(g_L, a^t, a^s, V_\pi) = (0, 1, 1, \text{any})$  forms a fixed line for four RGEs of  $g_L, a^t, a^s$  and  $V_\pi$ . When one point on this fixed line is selected through the matching procedure as done in Ref. 2, namely the value of  $V_{\pi,\text{bare}}$  is fixed, the present result implies that the point does not move in a subspace of the parameters. Approaching the restoration point of chiral symmetry, the physical pion velocity itself flows into the fixed point.

## 5. Matching Conditions on the Bare Pion Velocity

One possible way to determine the bare parameters is the Wilsonian matching proposed in Ref. 5 which is done by matching the axial-vector and vector current correlators derived from the HLS with those by the operator product expansion (OPE) in QCD at the matching scale  $\Lambda$ . In this section, we present the matching conditions to determine the bare pion velocity including the effect of Lorentz symmetry breaking at the bare level following Ref. 2.

In the EFT sector, pion couples to the longitudinal part of the axial-vector current correlator  $G_A^L$ . We regard  $G_A^{L,T}$  as functions of  $-q^2$  and  $|\vec{q}|^2$  instead of  $q_0$  and  $\vec{q}$ , and expand  $G_A^L$  in a Taylor series around  $\vec{q} = |\vec{q}| = 0$  in  $\vec{q}^2/(-q^2)$  as follows:

$$G_A^L(-q^2, \vec{q}^2) = G_A^{L(0)}(-q^2) + G_A^{L(1)}(-q^2)\vec{q}^2 + \cdots \quad (9)$$

Expanding the axial-vector current correlator derived from the HLS theory

$G_A^{(\text{HLS})L}$  in terms of  $\bar{q}^2/(-q^2)$ , we obtain

$$G_A^{(\text{HLS})L(0)}(-q^2) = \frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s}{-q^2} - 2z_{2,\text{bare}}^L, \quad (10)$$

$$G_A^{(\text{HLS})L(1)}(-q^2) = \frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s (1 - V_{\pi,\text{bare}}^2)}{(-q^2)^2}, \quad (11)$$

where  $z_{2,\text{bare}}^L$  is the parameter of the higher order term.

On the other hand, the axial-vector current correlator obtained from the OPE is given by<sup>17,18,19</sup>

$$\begin{aligned} G_A^{\mu\nu}(q_0, \bar{q}) &= (q^\mu q^\nu - g^{\mu\nu} q^2) \frac{-1}{4} \left[ \frac{1}{2\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{1}{6Q^4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_T \right. \\ &\quad - \frac{2\pi\alpha_s}{Q^6} \left\langle \left( \bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d \right)^2 \right\rangle_T \\ &\quad \left. - \frac{4\pi\alpha_s}{9Q^6} \left\langle \left( \bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d \right) \sum_q^{u,d,s} \bar{q}\gamma^\mu\lambda^a q \right\rangle_T \right] \\ &\quad + [-g^{\mu\nu} q^{\mu_1} q^{\mu_2} + g^{\mu\mu_1} q^\nu q^{\mu_2} + q^\mu q^{\mu_1} g^{\nu\mu_2} + g^{\mu\mu_1} g^{\nu\mu_2} Q^2] \\ &\quad \times \left[ \frac{4}{Q^4} A_{\mu_1\mu_2}^{4,2} + \frac{16}{Q^8} q^{\mu_3} q^{\mu_4} A_{\mu_1\mu_2\mu_3\mu_4}^{6,2} \right], \end{aligned} \quad (12)$$

where  $Q^2 = -q^2$ ,  $\tau = d - s$  denotes the twist, and  $s = 2k$  is the number of spin indices of the operator of dimension  $d$ . In the above expression, we restrict ourselves to contributions from the twist 2 ( $\tau = 2$ ) operators<sup>c</sup>.  $A_{\mu_1 \dots \mu_{2k}}^{2k+\tau, \tau}$  is the residual Wilson coefficient times matrix element of dimension  $d$  and twist  $\tau$ . The general tensor structure of the matrix element of  $A_{\mu_1 \dots \mu_{2k}}^{2k+\tau, \tau}$  is given in Ref. 17.

Now we proceed to estimate the pion velocity by matching to QCD. We require the following matching conditions at  $Q^2 = \Lambda^2$ :

$$\begin{aligned} Q^2 \frac{d}{dQ^2} G_A^{(\text{HLS})L(0)}(Q^2) &= Q^2 \frac{d}{dQ^2} G_A^{(\text{OPE})L(0)}(Q^2), \\ G_A^{(\text{HLS})L(1)}(Q^2) &= G_A^{(\text{OPE})L(1)}(Q^2). \end{aligned} \quad (13)$$

<sup>c</sup>The higher the twist of operators becomes, the more these operators are suppressed since the dimensions of such operators become higher and the power of  $1/Q^2$  appear.

They lead to the conditions on the bare pion decay constants as

$$\begin{aligned} \frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s}{\Lambda^2} &= \frac{1}{8\pi^2} \left[ \left(1 + \frac{\alpha_s}{\pi}\right) + \frac{2\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle_T}{\Lambda^4} + \pi^3 \frac{1408}{27} \frac{\alpha_s \langle \bar{q}q \rangle_T^2}{\Lambda^6} \right] \\ &\quad + \frac{\pi^2}{15} \frac{T^4}{\Lambda^4} A_{4,2}^\pi - \frac{16\pi^4}{21} \frac{T^6}{\Lambda^6} A_{6,4}^\pi \equiv G_0, \\ \frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s (1 - V_{\text{bare}}^2)}{\Lambda^2} &= \frac{32}{105} \pi^4 \frac{T^6}{\Lambda^6} A_{6,4}^\pi, \end{aligned} \quad (14)$$

where we use the dilute pion-gas approximation in order to evaluate the matrix element  $\langle \mathcal{O} \rangle_T$ <sup>17</sup> in the low temperature region. From these conditions, we obtain the following matching condition to determine the deviation of the bare pion velocity from the speed of light in the low temperature region:

$$\delta_{\text{bare}} \equiv 1 - V_{\pi,\text{bare}}^2 = \frac{1}{G_0} \frac{32}{105} \pi^4 \frac{T^6}{\Lambda^6} A_4^\pi. \quad (15)$$

This implies that the intrinsic temperature dependence starts from the  $\mathcal{O}(T^6)$  contribution. On the other hand, the hadronic thermal correction to the pion velocity starts from the  $\mathcal{O}(T^4)$  [see Eq. (7)]. Thus the hadronic thermal effect is dominant in low temperature region.

## 6. Pion Velocity near the Critical Temperature

In this section, we extend the matching condition valid at low temperature, Eq. (15), to near the critical temperature, and determine the bare pion velocity at  $T_c$ .

As is discussed in Ref. 2, we should in principle evaluate the matrix elements in terms of QCD variables only in order for performing the Wilsonian matching, which is as yet unavailable from model-independent QCD calculations. Therefore, we make an estimation by extending the dilute gas approximation adopted in the QCD sum-rule analysis in the low-temperature region to the critical temperature with including all the light degrees of freedom expected in the VM. In the HLS/VM theory, both the longitudinal and transverse vector mesons become massless at the critical temperature since the HLS gauge coupling constant  $g_L$  vanishes. At the critical point, the longitudinal vector meson which becomes the NG boson  $\sigma$  couples to the vector current whereas the transverse vector mesons decouple from the theory because of the vanishing  $g_L$ . Thus we assume that thermal fluctuations of the system are dominated near  $T_c$  not only by the pions but also by the longitudinal vector mesons. We evaluate the thermal matrix elements of the non-scalar operators in the OPE, by extending the thermal pion gas



approximation employed in Ref. 17 to the longitudinal vector mesons that figure in our approach. This is feasible since at the critical temperature, we expect the equality  $A_4^\rho(T_c) = A_4^\pi(T_c)$  to hold as the massless longitudinal vector meson is the chiral partner of the pion in the VM. It should be noted that, although we use the dilute gas approximation, the treatment here is already beyond the low-temperature approximation because the contribution from vector meson is negligible in the low-temperature region. Since we treat the pion as a massless particle in the present analysis, it is reasonable to take  $A_4^\pi(T) \simeq A_4^\pi(T = 0)$ . We therefore use

$$A_4^\rho(T) \simeq A_4^\pi(T) \simeq A_4^\pi(T = 0) \quad \text{for } T \simeq T_c. \quad (16)$$

Therefore from Eq. (15), we obtain the deviation  $\delta_{\text{bare}}$  as

$$\delta_{\text{bare}} = 1 - V_{\pi, \text{bare}}^2 = \frac{1}{G_0} \frac{32}{105} \pi^4 \frac{T^6}{\Lambda^6} [A_4^\pi + A_4^\rho]. \quad (17)$$

This is the matching condition to be used for determining the value of the bare pion velocity near the critical temperature.

Let us make a rough estimate of  $\delta_{\text{bare}}$ . For the range of matching scale ( $\Lambda = 0.8 - 1.1 \text{ GeV}$ ), that of QCD scale ( $\Lambda_{\text{QCD}} = 0.30 - 0.45 \text{ GeV}$ ) and critical temperature ( $T_c = 0.15 - 0.20 \text{ GeV}$ ), we get

$$\delta_{\text{bare}}(T_c) = 0.0061 - 0.29. \quad (18)$$

Thus we obtain the *bare* pion velocity as  $V_{\pi, \text{bare}}(T_c) = 0.83 - 0.99$ . Finally thanks to the non-renormalization property, i.e.,  $v_\pi(T_c) = V_{\pi, \text{bare}}(T_c)$  given in Eq. (8), we arrive at the physical pion velocity at chiral restoration:

$$v_\pi(T_c) = 0.83 - 0.99, \quad (19)$$

to be close to the speed of light.

## 7. Summary

In this talk, we started from the Lorentz non-invariant HLS Lagrangian at bare level and studied the pion velocity at the critical temperature based on the VM. We showed that *the pion velocity does not receive either quantum or hadronic corrections in the limit  $T \rightarrow T_c$* , which is protected by the VM. This non-renormalization property means that it suffices to compute the pion velocity at the level of bare HLS Lagrangian at the matching scale to arrive at the *physical* pion velocity at the critical temperature of chiral symmetry restoration. We derived the matching condition on the *bare* pion velocity and finally we found that the pion velocity near  $T_c$  is close to the speed of light,  $v_\pi(T) = 0.83 - 0.99$ .

This is in contrast to the result obtained from the chiral theory<sup>20</sup>, where the relevant degree of freedom near  $T_c$  is only the pion. Their result is that the pion velocity becomes zero for  $T \rightarrow T_c$ . Therefore from the experimental data, we may be able to distinguish which picture is correct,  $v_\pi \sim 1$  or  $v_\pi \rightarrow 0$ .

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